INTERNATIONAL BACCALAUREATE

Mathematics: applications and interpretation

MAI

EXERCISES [MAI 2.1]

LINES

Compiled by Christos Nikolaidis

A. Paper 1 questions (SHORT)

1. [Maximum mark: 9]

Consider the points $A(2,7)$, $B(5,11)$

(a) Find the gradient of the line segment $[AB]$. [2]

(b) Write down the gradient of a perpendicular line to (AB). [1]

(c) Let M be the midpoint M of the line segment $[AB]$. Find the coordinates of M. [2]

(d) Find the distance between A and B (i.e. the length AB). [2]

(e) Given that B is the midpoint of $[AC]$, find the coordinates of C [2]

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2. [Maximum mark: 8]

Consider the points $A(3,5)$, $B(a,1)$, $C(7,-3)$ where $a \in R$.

Find the possible values of a in each of the following cases:

(a) If the gradient of the line (AB) is 2. [2]

(b) If the midpoint of the line segment [AB] is $M(7,3)$. [2]

(c) If B is the midpoint of the line segment $[AC]$. [1]

(d) If the distance between A and B is 5. [3]

3. [Maximum mark: 9]

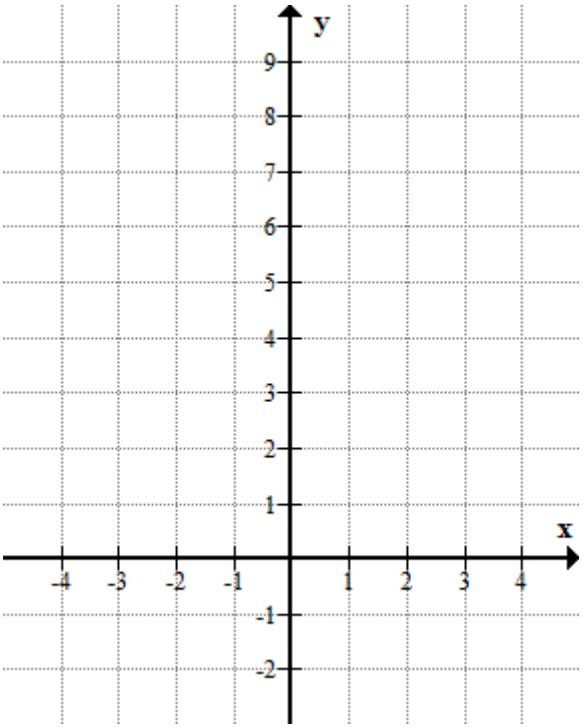
Consider the line L given by $y = 2x + 4$

(a) Write down

(i) the gradient of the line (ii) the y-intercept (iii) the x-intercept [3]

(b) Draw the line on the diagram below. [3]

(c) Check if the points A(7,19) and B(8,20) lie on the line. [3]



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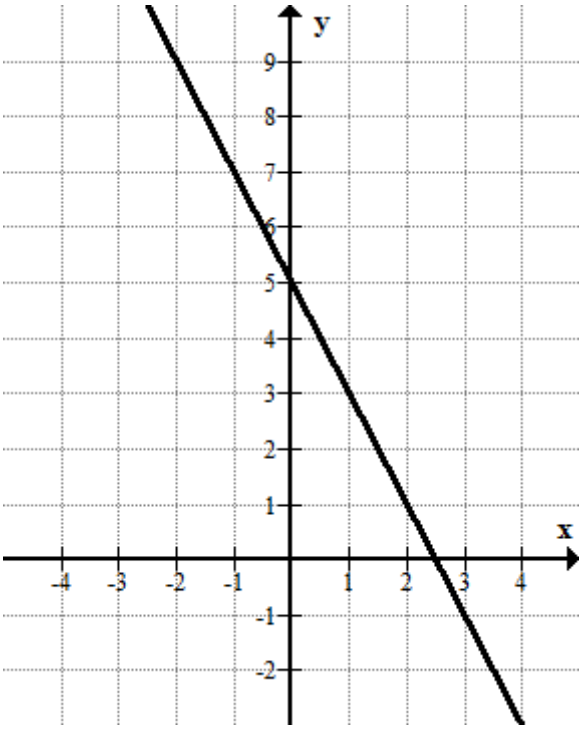
4. [Maximum mark: 9]

Consider the line L on the diagram below



(a) Write down

(i) the gradient of the line (ii) the y-intercept (iii) the x-intercept [3]

(b) Write down the equation of the line in the gradient-intercept form $y = mx + c$ [2]

(c) Given that P(1, y) and Q(x,1) lie on the line write down the values of x and y. [2]

(d) Given that A(a,– 5) and B(-5,b) lie on the line find the values of a and b. [2]

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5. [Maximum mark: 7]

Find the equation of the line passing through A(3,4) and B(5,7)

(a) in the gradient-point form $y - y_1 = m(x - x_1)$ [3]

(b) in the gradient-intercept form $y = mx + c$. [2]

(c) in the form $ax + by = d$, where a, b, d are integers. [2]

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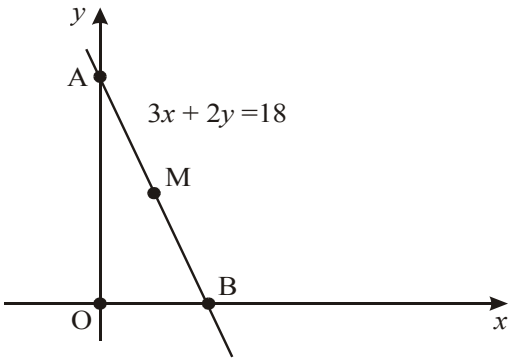
6. [Maximum mark: 6]

The diagram below shows the line with equation $3x + 2y = 18$. The points A and B are the y and x-intercepts respectively. M is the midpoint of [AB].



Find the coordinates of (i) the point A; (ii) the point B; (iii) the point M.

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7. [Maximum mark: 8]

Consider the line L with equation $y + 2x = 3$. The line L_1 is parallel to L and passes through the point $(6, -4)$.

(a) Find the gradient of L_1. [1]

(b) Find the equation of L_1 in the form $y = mx + b$. [3]

(c) Find the x-coordinate of the point where line L_1 crosses the x-axis. [2]

(d) Draw the two lines on the diagram below. [2]

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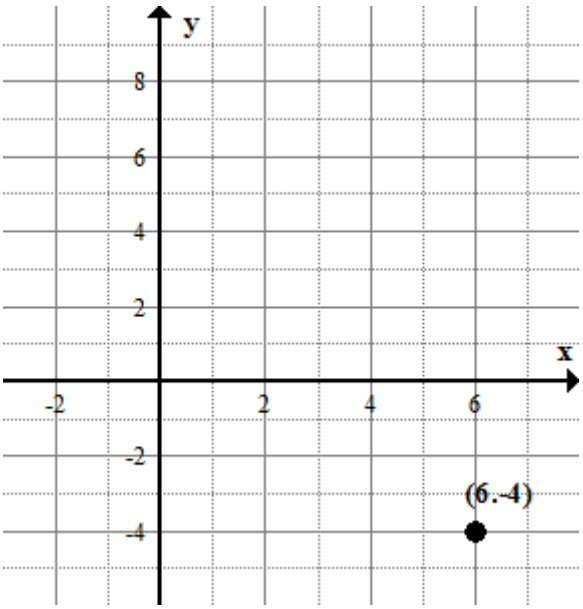

8. [Maximum mark: 5]

(a) Find the equation of the line passing though the points $A(2,-5)$ and $B(2,8)$. [2]

(b) Find the equation of the line passing though the points $C(6,5)$ and $D(-3,5)$. [2]

(c) Find the point of intersection P between the lines L_1 and L_2 [1]

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9. [Maximum mark: 6]

(a) Find the equation of the line which is parallel to x-axis and passes through A(2,3) [2]

(b) Find the equation of the line which is parallel to y-axis and passes through A(2,3) [2]

(c) Find the equation of the line passing though the origin and A(2,3) [2]

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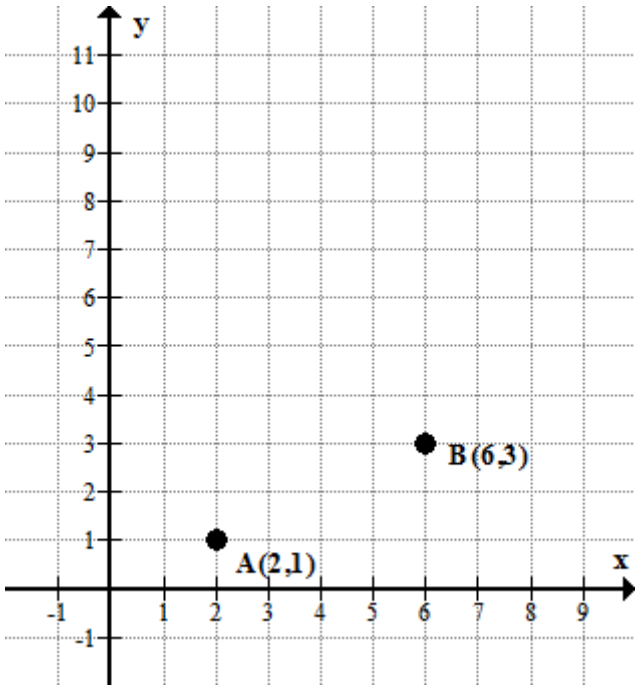
10. [Maximum mark: 7]

The points A(2,1) and B(6,3) are shown in the diagram below.



Let L, be the **perpendicular bisector** of the line segment [AB]

(a) Find the equation of L. [5]

(b) Write down the y-intercept of L and draw an accurate line for L on the diagram above. . [2]

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11. [Maximum mark: 9]

Let $\mathrm{A}(3k,4k)$ be a point of the line L with equation $y=\frac{4}{3}x$, where k is an integer.

(a) Verify that the point A lies on the line L. [2]

(b) Find the possible values of k if the distance between the origin and A equals 10. [3]

(c) Write down the coordinates of the two points on the line L whose distance from the origin is equal to 10. [2]

(d) Demonstrate on the diagram below the result of question (c). [2]

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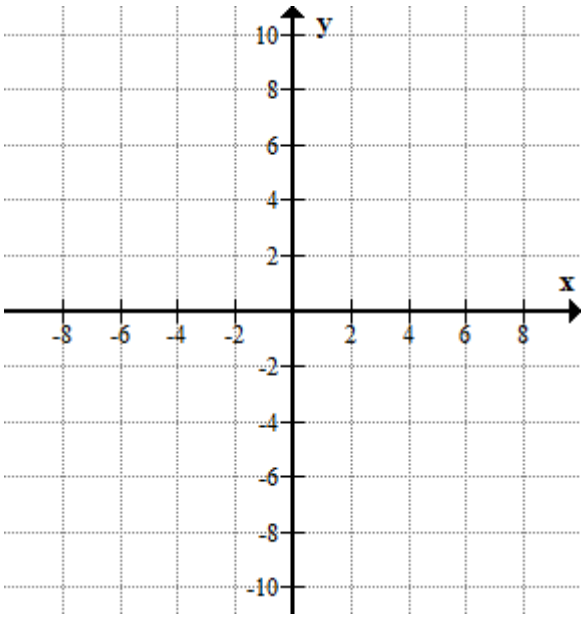

B. Paper 2 questions (LONG)

12. [Maximum mark: 12]

Consider the line L_1 with equation $y = 2x - 3$.

The line L_2 is **parallel** to L_1 and passes through the point $\mathrm{A}(1,9)$.

The line L_3 is **perpendicular** to L_1 and passes through the point $\mathrm{A}(1,9)$.

(a) Find the equation of the line L_2. [3]

(b) Find the equation of the line L_3. [3]

The lines L_1 and L_3 intersect at point B

(c) Find the coordinates of point B. [2]

(d) Find the distance between the points A and B. [2]

(e) Sketch a diagram and deduce the distance from the point A to the line L_1. [2]

[it is not necessary to draw the Cartesian plane]

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13. [Maximum mark: 10]

Consider the points $\mathrm{A}(2,5)$ and $\mathrm{B}(3,8)$.

(a) Find the gradient of the line L_1 passing through A and B. [2]

(b) Express in the form $ax + by = c$, where a, b, c are integers

(i) the equation of the line L_1 .

(ii) the equation of the line L_2 which is perpendicular to L_1 and passes through the point A. [5]

(c) Write down the solution of the two simultaneous equations found in (b). [1]

(d) State what the solution in question (c) represents; Make a comment by sketching the two lines L_1, L_2 and the solution in question (c). [2]

[it is not necessary to draw the Cartesian plane]

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14. [Maximum mark: 10]

Consider the points $A(-2,5)$ and $B(4,9)$.

(a) Find the gradient of the line L passing through A and B. [2]

(b) Find the coordinates of the midpoint M between A and B. [2]

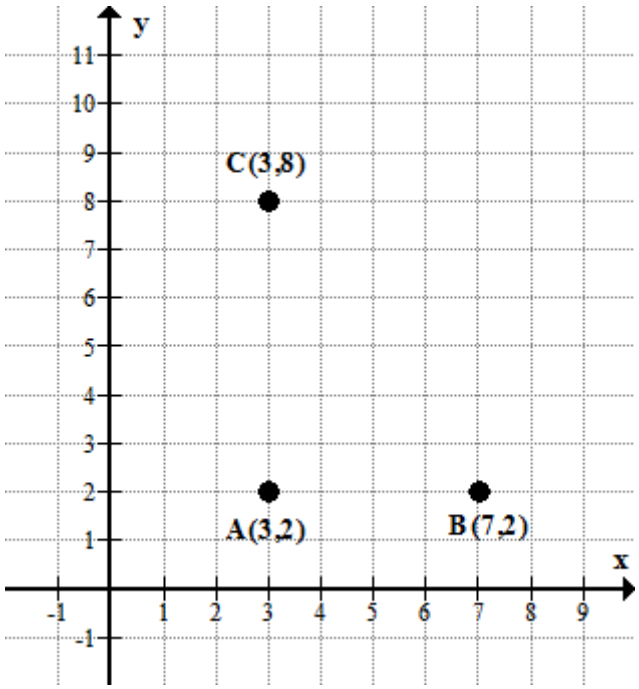
(c) Find the equation of the line which is perpendicular to L and passes through the point M (i.e. the **perpendicular bisector** of the line segment [AB]) [2]

(d) Find the distance between the points A and B. [2]

(e) Write down the distance between M and B, in the form $\sqrt{a}$, where $a \in Z$. [2]

15. [Maximum mark: 23]

The points A(3,2), B(7,2) and C(3,8) are shown in the diagram below.



(a) Find the equation of the perpendicular bisector of line segment [AB]. [2]

(b) Find the equation of the perpendicular bisector of line segment [AC]. [2]

(c) Write down the coordinates of the point of intersection P of the two bisectors and show that P is the midpoint of the line segment [BC]. [3]

(d) Find the areas of the triangles

(i) ABC. (ii) ABP (iii) ACP [6]

(e) Find the equation of the perpendicular bisector L of the line segment [BC] in the form $ax + by + d = 0$ with $a, b, c \in Z$. [5]

(f) Show that the line L does not pass through A. [2]

(g) Draw the three perpendicular bisectors of the sides of ABC on the diagram above. [3]

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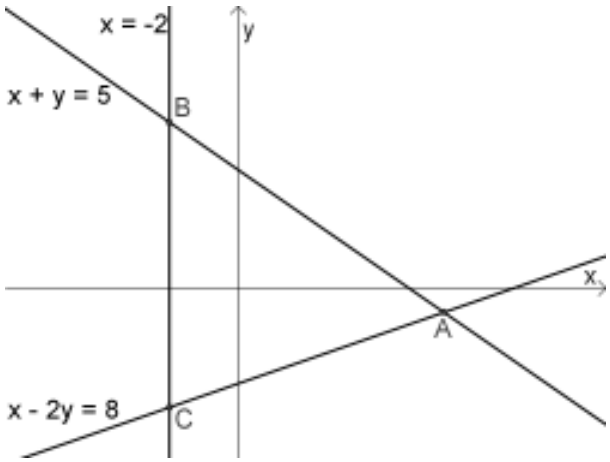
16. [Maximum mark: 10]

The following three lines l_1, l_2, and l_3 are defined with equations

$$l_1 : x + y = 5, \qquad l_2 : x - 2y = 8, \qquad l_3 : x = -2$$

and are shown in the figure below.



(a) Find the coordinates of the common point A between the lines l_1 and l_2. [2]

(b) Write down the coordinates of the common point B between the lines l_1 and l_3 [2]

(c) Write down the coordinates of the common point C between the lines l_2 and l_3. [2]

(d) **Hence**, find the area of the triangle ABC. [4]

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